Tanner Dean's Scholars Summer Grant Abstract and Biographical Sketch

Vivek A. Pisharody, Class of 2018

Mathematics and Physics

Modern mathematics in theoretical physics

Abstract

The subtle interplay of mathematics and physics has a long and noted history. This is perhaps truest in the present era, where the highest realms of mathematics have permeated theoretical physics, whilst notions from physics have suffused mathematics. Unfortunately, while Cornell has excellent faculties and classes in both departments, there is little coursework which provides an understanding of current approaches to physical theory in the argot of mathematicians.

I think I now have the requisite mathematical and physical background to begin studying this realm in earnest. The primary requirement at this point is merely time, which is difficult to come by during the year. I propose to use the Tanner Dean's Scholars Summer Grant to remain at Cornell this summer to study current mathematical approaches to theoretical physics.

The work undertaken this summer will produce part of a planned mathematics undergraduate thesis and will also be an essential part of my preparation for graduate studies in mathematics.

Biographical Sketch

I am a Junior in the College of Arts and Sciences majoring in Mathematics and Physics. My other academic interests include Chinese language and history. I am the LGBTQ+ Liason for the Society of Physics Students, Co-President of OStem@Cornell ("Out in STEM at Cornell"), and a member of the senior honor society Der Hexenkreis.

Tanner Dean's Scholars Summer Grant Statement of Purpose

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Statement of Purpose

The aim of my project is to begin studying the mathematics of theoretical physics in earnest, with an eye toward completing a senior thesis in mathematics. This summer, I want to begin studying Alain Conne's treatise on non-commutative geometry and physics along with some relevant papers. I have read portions of L. Takhtajan's Quantum Mechanics for Mathematicians and Quantum Fields and Strings for Mathematicians by P. Deligne et al., which, combined with my mathematics and physics coursework, should prepare me for this project.

However, as I am currently studying *Lie Groups Beyond an Introduction* by Anthony Knapp in a reading course with Professor B. Speh, I plan to begin this summer project by reading the "special topics" sections at the end of Knapp's book. This material is all quite useful to know for theoretical physics. In particular, I will spend the first two to three weeks on Chapters VII (*Advanced Structure Theory*), IX (*Induced Representations and Branching Theorems*), and X (*Prehomogeneous Vector Spaces*). The discussion of branching theorems in Chapter IX is of particular importance for mathematical physics.

After completing this section, I will turn my attention to Connes' text *Noncommutative Geometry*. remains near the forefront of current research in the field. Work in the vein of Connes' text, it is thought, may provide important mathematical context for new physics in the twenty-first century. I hope to read the first three to four chapters during the remainder of the summer. Throughout the project, I will produce weekly writeups of my progress and of the problems and exercises solved.

Planning for Contingencies

As with any plan, derailments are certainly possible. A text may prove more challenging than anticipated, taking an excess of time or even requiring a digression through other, more elementary texts.

In the event that Knapp's text consumes more time than anticipated, I can devote the remainder of the summer to other texts on Lie theory instead of Connes' text. I would then consult with Professors Speh and Knutson to select important papers in the field. In the event that my attempt to read Connes' text proves premature, I will consult one of the texts listed below (Raeburn and Williams, Brown and Ozawa, or Rordam and Stormer) to further develop my background before returning to Connes.

Regardless, the basic structure of the project and its deliverables are unchanged. Even in the event of such a derailment, I will still produce weekly reports and a final paper, as discussed in the next section.

Project timeline and outcomes

I expect to be on campus between approximately 5 June and 15 August (10 weeks).

I aim to produce weekly reports containing (1) an expository report on the texts read and (2) the problems and exercises solved. I will prepare an introductory exposition on the material studied by the beginning of fall semester. The exposition written this summer will be the basis of my senior thesis in mathematics.

Budget calculations

The expected expenditures for this project are modest. The primary expenses will be summer lodging and food, and the texts themselves. Without Tanner Dean's Scholars funding, I will not be able to solely devote this summer to my studies; moreover, I certainly will not be able to remain on campus and consequently, would lose access to some of our library system's physical resources.

I have attached at the end of this document the completed budget worksheet; below, I have outlined my calculations. My lease includes summer housing at a rate of \$603 per month. Last year, Cornell Dining priced weekday meals at \$16.63 for dinner and \$12.10 for lunch. I would consider a Cornell Dining plan, but summer plans information has not been released, and last year's plans only covered mid-June through early August. Based on this data, I will budget food expenses at \$100 per week.

Texts and other resources for are expected to run approximately \$400 based on prices currently listed online. A more complete breakdown of some text prices is listed below. Some texts are are not listed because they are out of print and/or available freely. I have sought library copies when possible.

Table 1: Estimated Book Prices

Text	\mathbf{Cost} (\$)
Lie Groups Beyond an Introduction. (Knapp)	89.00
Quantum Mechanics for Mathematicians. (L. Takhtajan)	60.00
Quantum Fields and Strings: A Course for Mathematicians. (Deligne et al.)	72.00
Quantum Field Theory: A Tourist Guide for Mathematicians. (Folland)	92.20
Classification of Nuclear C*-Algebras. (M. Rordam, E. Stormer)	104.98
Morita Equivalence and Continuous-trace C*-algebras. (I. Raeburn, D.P. Williams.)	32.00
Total	\$450.18

My housing is located relatively far from campus, I would like to purchase a bus pass if funding permits. Based on the information from the TCAT website, a Zone 1 bus pass for 65 days costs \$97.50. Based on the above outline, the estimated budget is summarized below:

	Table 2: Budget			
	Estimated Rate	Estimated '	Total Cost (\$)	
Housing	\$603 per month	1206		
Meals	\$100 per week	1000		
Texts		400		
Bus pass		97.00		
Total		\$2,803.50	-, 2,804	

I have attached the budget worksheet at the very end of this application. Below is a list of the texts I referred to in producing this application, and which I will use in the course of the project.

References

- [1] N.P. Brown and N. Ozawa. C^* -algebras and Finite-dimensional Approximations. Graduate studies in mathematics. American Mathematical Society, 2008. ISBN: 9780821843819.
- [2] A. Connes. Noncommutative Geometry. Dunod, 2005. ISBN: 9782100493722.
- [3] P. Deligne. Quantum Fields and Strings: A Course for Mathematicians. Quantum Fields and Strings: A Course for Mathematicians v. 1. American Mathematical Society, 1999. ISBN: 9780821820124.
- [4] G.B. Folland. Quantum Field Theory: A Tourist Guide for Mathematicians. Mathematical surveys and monographs. American Mathematical Society, 2008. ISBN: 9780821847053.
- [5] A.W. Knapp. *Lie Groups Beyond an Introduction*. Progress in Mathematics. Birkhauser Boston, 2002. ISBN: 9780817642594.
- [6] I. Raeburn and D.P. Williams. *Morita Equivalence and Continuous-trace C*-algebras*. Mathematical surveys and monographs. American Mathematical Society, 1998. ISBN: 9780821808603.
- [7] M. Rordam and E. Stormer. Classification of Nuclear C*-Algebras. Entropy in Operator Algebras. Encyclopaedia of Mathematical Sciences. Springer Berlin Heidelberg, 2001. ISBN: 9783540423058.
- [8] L.A. Takhtajan. Quantum Mechanics for Mathematicians. Graduate Studies in Mathematics. American Mathematical Society, 2008. ISBN: 9780821846308.